Lab 3

Introduction to Stereographic Projection

In this experiment, the aim is to provide a practical and theoretical introduction to the stereographic projection in order to use it in morphological crystallography of polycrystalline materials.

The stereographic projection is a projection of points from the surface of a sphere on to its equatorial plane and it provides a useful way to conveying information about the orientation of lines and planes in 3-dimensional (3D) space. Stereographic projection can be defined as a graphical technique for representing the angular relationships between planes and directions in crystals on a 2D piece of paper.

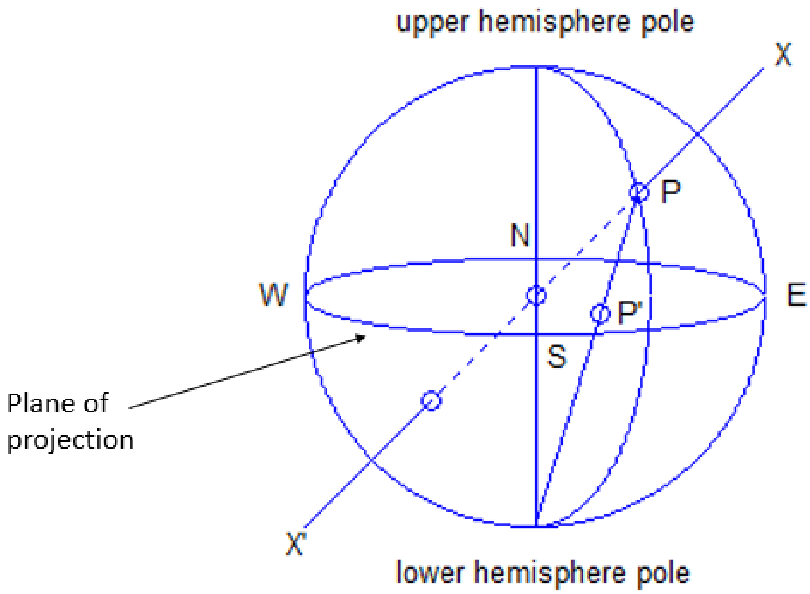


Fig.3.1 The stereographic projection of the line X-X'.

For any X-X', a sphere can be constructed with its origin centered on the line, see Fig.3.1. The line must intersect this sphere at two points: one being in the upper hemisphere and the other being in the lower hemisphere. An exception is a horizontal line, which intersects the equator twice. Now, construct a new line connecting the intercept on upper hemisphere (point P) and the lower hemisphere pole. This new line will intersect the horizontal plane that passes through the origin of the sphere (i.e. plane of projection), at one unique point (P'). The position of this point (P') only depends on the orientation of the line X-X'. Point P' represents the stereographic projection of the line X-X'. The plane of projection viewed from directly above is given in Fig.3.2.

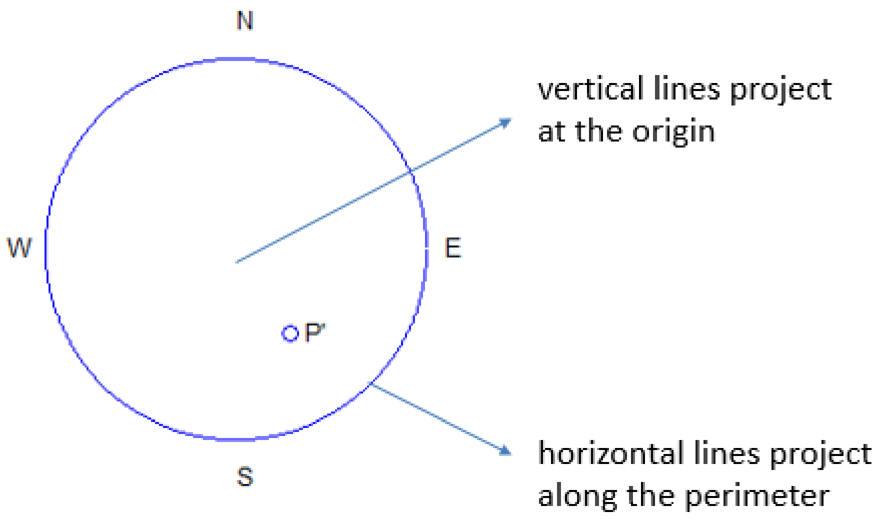


Fig. 3.2 Top view of the projection plane.

Planes can be treated in a similar fashion. However, in that case the projection will be a line (called as "great circle") instead of a point, see Fig.3.3 and Fig.3.4.

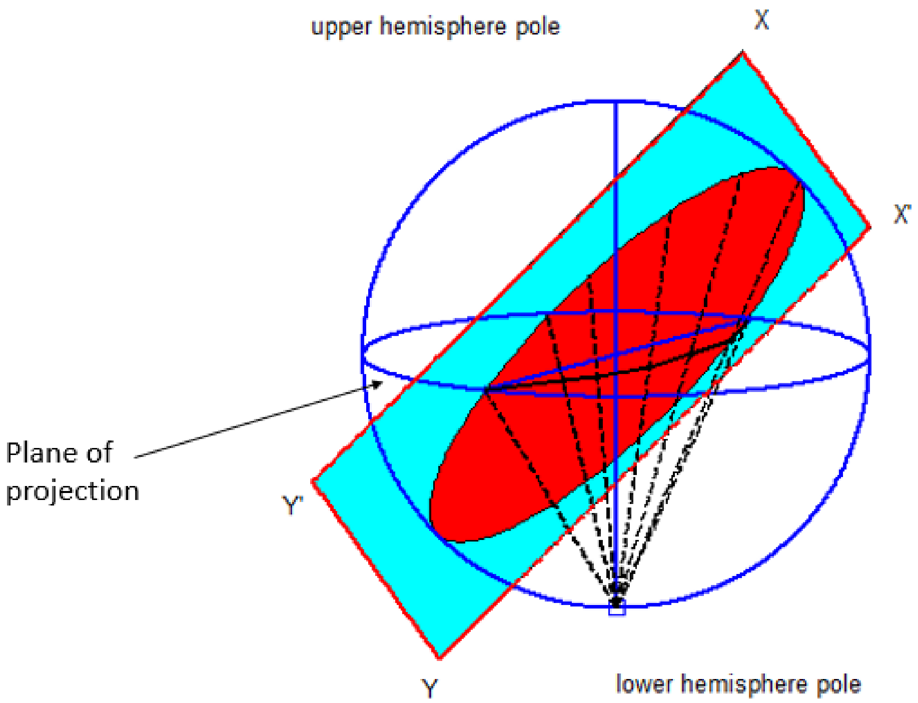


Fig.3.3 The projection of the plane X-X'-Y-Y'

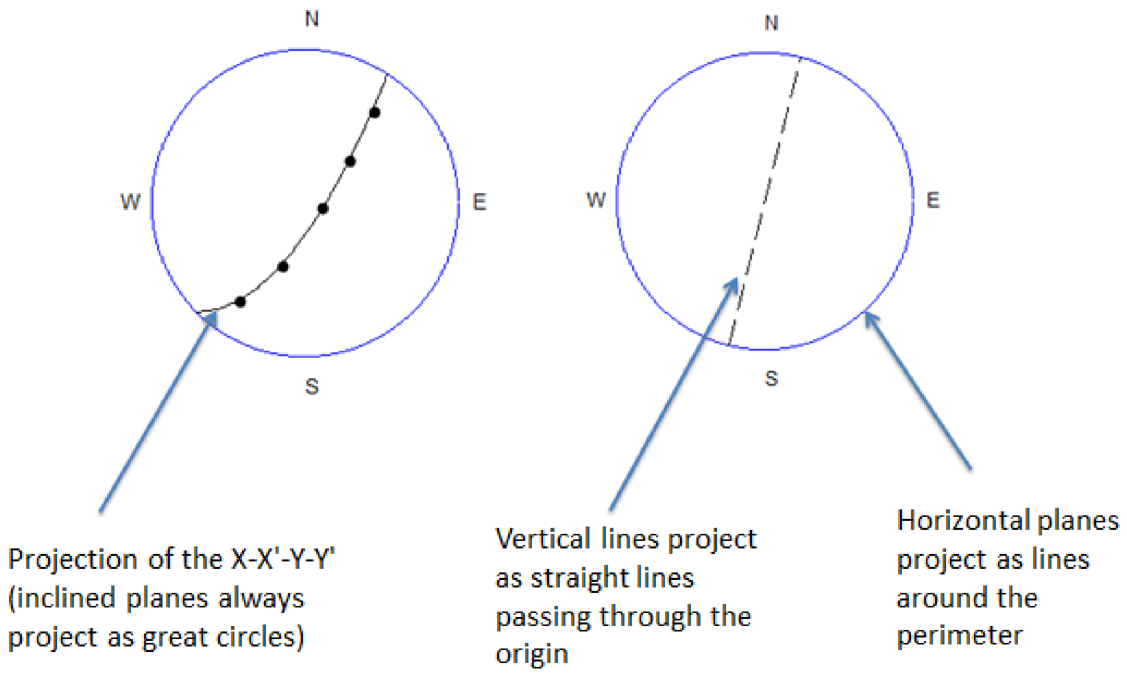


Fig.3.4 Top view of the projection plane.

The stereographic projection provides a powerful graphical method for conveying quantitative information about the orientation of crystal faces as well as the symmetry elements Also, note that, when dealing with crystal faces it is convenient to plot the “pole” of the direction, which is normal to the crystal face rather than the plane itself.

Lab 4

Stereographic Projection

Stereographic projection can be defined as a graphical technique for representing the angular relationships between planes and directions in crystal on a 2D piece of paper.

4.1. Importance of the Stereographic Projection

Stereographic projection is important since directions in three-dimensional (3D) space can be represented fully as a set of points on the surface of the sphere. The Stereographic projection of these points is the best way of representing the inter-relationships of a set of directions on a flat piece of paper.

The stereographic projection is used to represent the angles between the faces of a crystal and the symmetry relationships between them. Imagine that the crystal is centered within a sphere, the normal to the crystal faces will give a consistent set of points uninfluenced by the relative sizes of the faces. The symmetry of the arrangement of these points on the sphere reveals the symmetry of the crystal. This symmetry can also be recognized in the stereographic projection of these points.

GREAT CIRCLE: A circle centered on the center of a sphere, with the same radius as the sphere.

SMALL CIRCLE: A circle not centered on the center of a sphere, has a smaller radius than the sphere.

Remember that in the cubic system, any direction [hkl] is perpendicular to the plane (hkl).

Interplanar angles in cubic system:

- The angle between the two normal to planes ($h_1k_1l_1$) and ($h_2k_2l_2$), or
- The angle between to planes (($h_1k_1l_1$) and ($h_2k_2l_2$), or
- The angle between directions [$h_1k_1l_1$] and [$h_2k_2l_2$], in cubic lattices is given as follows:

$$\cos\theta = \frac{a.b}{ab} = \frac{h_1h_2 + k_1k_2 + l_1l_2}{\sqrt{(h_1^2 + k_1^2 + l_1^2)(h_2^2 + k_2^2 + l_2^2)}}$$

In cubic lattices, the direction [hkl] is perpendicular to the plane (hkl), and the angle between two planes is equal to the angle between two normal to these planes.

4.2. Spherical Projection

A crystal is positioned with its center at the center of a sphere. This sphere is called as “reference sphere” or “sphere of projection”. Crystal planes and directions can be represented on the surface of this sphere.

Representation of planes:

- Draw the normal to a plane in the crystal through the center of the sphere to intersect the sphere surface at a point called “pole”, or
- Extend the plane in the crystal trough the center of the sphere to intersect the surface of the sphere. The plane intersect the sphere surface to give a “great circle”.

Angle between planes:

Consider two planes “a” and “b” in a crystal at the center of the projection sphere. The planes can be represented as great circles “A” and “B” on the sphere surface. The normal to these planes, “OP” and “OQ”, Intersect the sphere at nodes “P” and “Q” on the sphere. The angle “α” between the planes and the normal to the planes is represented by the distance “PQ” on the great circle passing through the nodes “P” and “Q”. ıf this great circle is calibrated in degrees, then the angle between the planes can be determined directly. To achieve this, a calibrated reference grid is needed.

The “spherical projection of the crystal” is given in Figure 4.1. In order to have a projection in two-dimension (2D), i.e. on a flat piece of paper, “stereographic projection” must be considered.

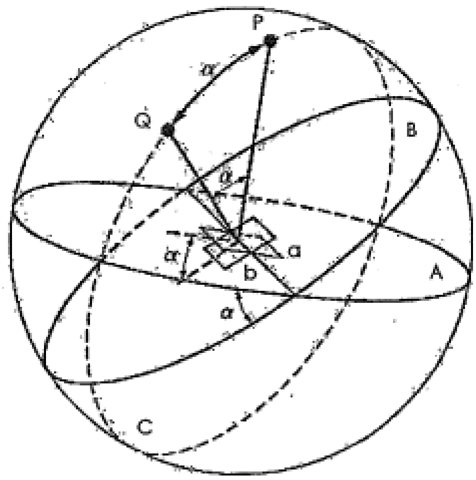


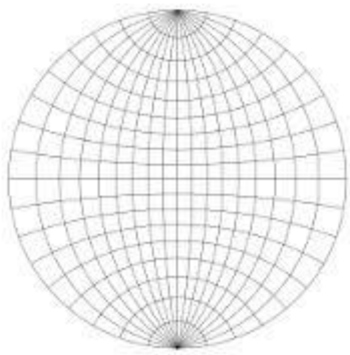
Figure 4.1. Spherical projection of the crystal

Representation of directions (or zone axis);

Crystal directions, which are represented by lines passing through the center of the sphere, intersect the sphere at wo nodes.

4.3. Wulff Net

In order to measure angles between projected poles in a stereographic projection, it is necessary to project the calibrated reference grid of lines of latitude and longitude onto a central plane. The most obvious projection is shown at the left below. Lines of latitude project to concentric circles and lines of longitude project to diameters of the primitive circle. However, this is not the most useful projection.



If the sphere is tilted such that the North South axis becomes horizontal, then the projection shown at the right in the above figure is called as the "Wulff net". In this projection, lines of latitude on the reference sphere project to the horizontal arcs shown. The lines of longitude (great circle) project to the vertical arcs from the North Pole to South Pole. In most cases, a

Wulff net is used since one needs to rotate the reference grid about an axis parallel to the plane of its equator to bring any two poles onto the same great circle.

The stereographical projection is usually drawn with the same diameter as a Wulff net printed on a transparent sheet. The projection and net are rotated relative to each other about their centers until the poles lie on the same projected great circle. The net is usually calibrated to 2° or 5° intervals. Angles between poles can be measured by counting the number of lines of projected latitude between them.

4.4. Projection of Important Directions And Planes Of A Cubic Crystal

It is often convenient to use a crystal in a standard orientation as a reference. For this standard orientation, a crystal with one set of cube planes parallel to the projection direction is usually taken.

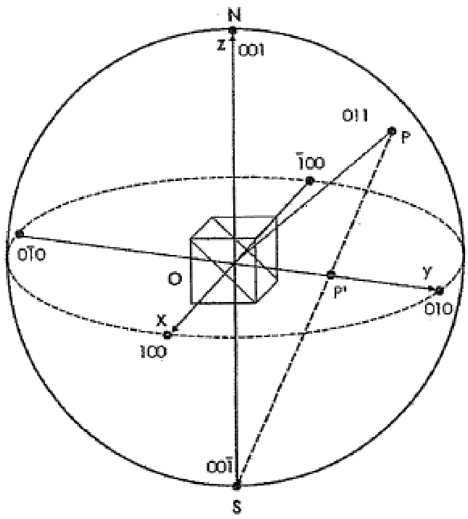
Consider three important groups of general indices:

<100> cube edges

<110> face diagonals

<111> body diagonals

Imagine a crystal at the center of the reference sphere with its [001] direction coincident with the N-S axis of the sphere. The [100] and [010] directions will lie on the equatorial plane. Try to relate the orientation of each plane to the position of its pole in the spherical projection below. The (011) plane is shown below;



The northern hemisphere is projected onto the equatorial plane to obtain the stereographic projection. This is called as the “001 standard projection of a cubic crystal”. Only the information from the northern hemisphere is needed since the southern hemisphere is a duplicate with negative indices.

This projection shows the poles of the cubic system. Remember that, in the cubic system the direction [hkl] is perpendicular to the plane (hkl). In the projections for the cubic system, one can omit the brackets while indexing. The hkl indexing may represent the direction index [hkl] as well as the normal to the plane (hkl) and hence the plane itself.

4.5. Plotting the 001 Standard Projection

Planes of a zone are planes that are all parallel to one line, called the zone axis, and the zone, i.e. the set of planes, is specified by giving the indices of the zone axis. Such planes may have quite different indices and spacing, the only requirement being their parallelism to a line. If the axis of a zone has indices [uvw], then any plane belongs to that zone whose indices {hkl} satisfies the relaxation:

Normal to planes belonging to one zone are coplanar and at right angles to the zone axis. The poles of planes of the same zone all lie on the same great circle, the zone axis is the pole of this trace (great circle).

EXAMPLE:

It is known that the 011 pole is in the zone of the 100 zone axis. The poles of all the planes of the 100 zone axis lie on the trace defined by this pole. The trace of this pole is the lie joining up 010, 001, and 010. Therefore, 011 projects somewhere between 001 and 010. From the interplanar relationships given below, it is known that the angle between {011} and {100} is 45°. Hence, by using a Wulff net, the 011 plane can be located 45° from 001.

The trace of the zone of the (011) plane can be drawn on the projection by drawing the great circle associated with the 011 pole.

4.6 Manipulations of the Stereographic Projection

Measuring the angle between two poles

1. Rotate the stereographic projection and the Wulff net relative to each other about their common centers until both poles lie on the same great circle. The poles may not lie exactly on the same great circle, but it is easy to align the poles so that they are exactly the same distance from the nearest great circle on the net.
2. The angle can be directly determined by counting the calibrations on the great circle.

Plotting the trace of a plane (If the pole representing its normal has been plotted previously)

1. Rotate the projection until the pole lies on the equator of the Wulff net.
2. Count 90° along the equator passing through the center of the net and mark the point
3. The great circle on the net, which passes through this point, gives the trace of the pole.

Drawing another projection after the crystal has been rotated

1. Decide which pole will be in the center of the projection after the crystal has been rotated.
2. Rotate the projection until this pole lies on the equator of the Wulff net and count the number of degrees between the pole and the equator along the equator.
3. Move all the poles by that number of degrees in the same direction along their own small circles.
4. Poles that move off the net are dropped. Poles, which are moved onto the projection, can usually be indexed by symmetry or by a vector consideration.

The 001 standard projection of a cubic crystal

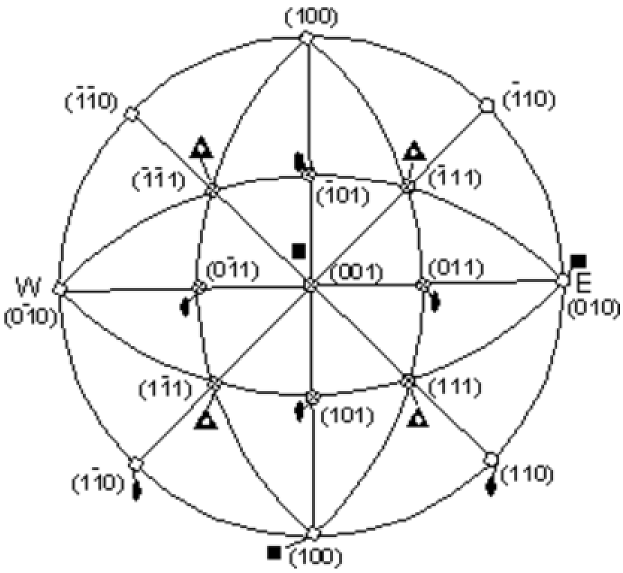


Figure 4.2 The 001 standard projection of a cubic crystal: a useful aid as it shows clearly the relative presentation of all the important plans in a crystal.

<u>CLASSWORK: Exercises on Stereographic Projection</u>

A cubic single crystal with side planes as shown below is cut along the lines indicated. What are the indices of the plane exposed by this cutting?